MMAT5360 200316

Bimatrix game (NTU)

Cooperative game with non-transferable utility

Nash bargaining solution (TU)

Cooperative game with transferable utility

2-person game

Threat solution

Def. (A, B) : game bimatrix

1. $T = A - B$: threat matrix

2. $\delta = v(T) = v(A-B)$: threat differential

3. maximin strategy $\vec{p}_d$ of T \
   minimax strategy $\vec{q}_d$ of T : threat strategies

4. $(\mu_d, \nu_d) = (\vec{p}_d A \vec{q}_d^{\,\prime}, \vec{p}_d B \vec{q}_d^{\,T})$: threat point
disagreement point

Note: $\mu_d - \nu_d = \delta$

5. $(\varphi_1, \varphi_2) = \left(\frac{\sigma+\delta}{2}, \frac{\sigma-\delta}{2}\right) = \left(\frac{\sigma+\mu_d-\nu_d}{2}, \frac{\sigma-\mu_d+\nu_d}{2}\right)$

threat solution

$\sigma = \max_{i,j}(a_{ij} + b_{ij})$ maximum total payoff

Note: $\begin{cases} \varphi_1 + \varphi_2 = \sigma \\ \varphi_1 - \varphi_2 = \delta \end{cases}$

Example: $(A, B) = \begin{pmatrix} (2,8) & (7,5) \\ (3,-1) & (-2,6) \end{pmatrix}$

Nash equilibrium: Player I: $\vec{p}_B = (0.7, 0.3)$

Player II: $\vec{q}_A = (0.9, 0.1)$

Payoff : $(2.5, 5.3)$

$$T = A - B = \begin{array}{cc} \begin{pmatrix} -6 & 2 \\ 4 & -8 \end{pmatrix} & \begin{array}{c} -8 \\ 12 \end{array} \\ \begin{array}{cc} -10 & 10 \\ 0.5 & 0.5 \end{array} & \end{array} \quad \begin{array}{c} 0.6 \\ 0.4 \end{array}$$

$$\vec{p}_d = (0.6, 0.4), \quad \vec{q}_d = (0.5, 0.5)$$

$$\delta = -2$$

$$(\mu_d, \nu_d) = (\vec{p}_d A \vec{q}_d^{\,T}, \vec{p}_d B \vec{q}_d^{\,T}) = (2.9, 4.9)$$

$$\sigma = 7 + 5 = 12$$

Threat solution: $(\varphi_1, \varphi_2) = \left(\frac{\sigma+\delta}{2}, \frac{\sigma-\delta}{2}\right)$

$$= \left(\frac{12-2}{2}, \frac{12+2}{2}\right)$$

$$= (5, 7)$$

Example 2: $(A, B) = \big((5,3) \ (4,5) \ \cancel{(3,2)} \big)$

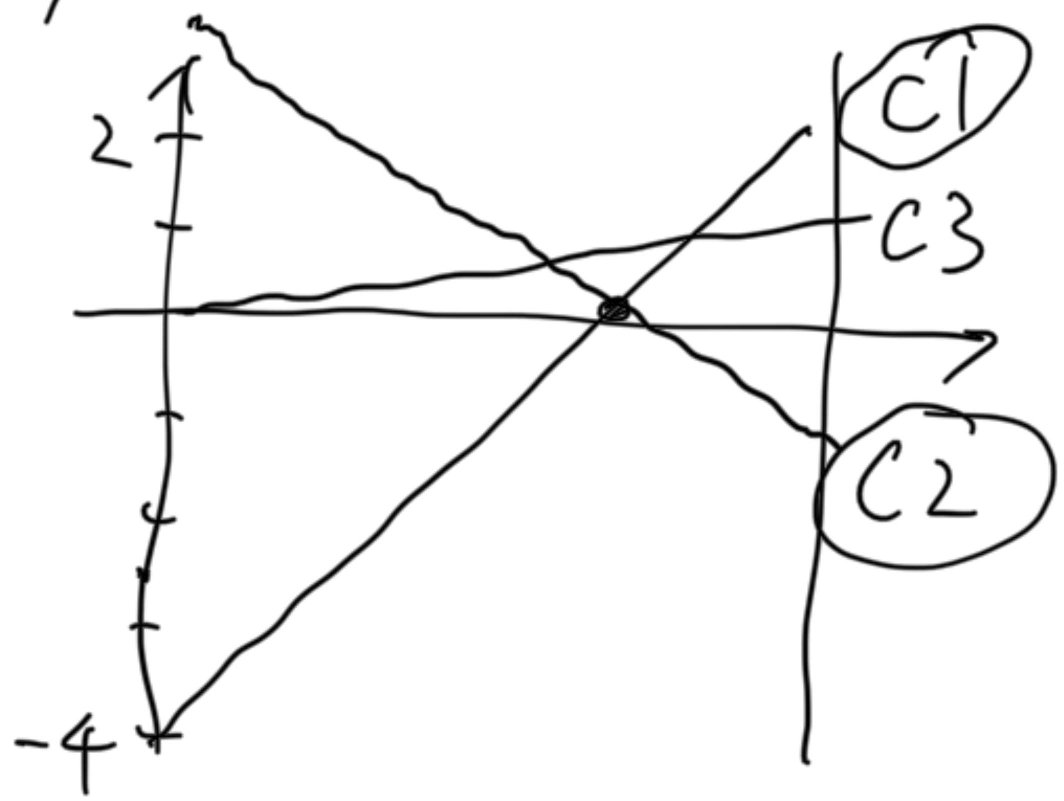
$(2,6)\ (6,3)\ \cancel{(4,4)}$

Nash equilibrium: $\vec{p}_B = (0.6, 0.4)$

$\vec{q}_A = (0.4, 0.6, 0)$

Payoff: $(4.4, 4.2)$

$$T = \begin{pmatrix} 2 & -1 & 1 \\ -4 & 3 & 0 \end{pmatrix}$$



$\vec{p}_d = (0.7, 0.3)$

$\vec{q}_d = (0.4, 0.6, 0)$

$\delta = v(T) = 0.2$

$\sigma = 9$

Threat solution: $(\varphi_1, \varphi_2) = \left(\frac{9+0.2}{2}, \frac{9-0.2}{2}\right) = (4.6, 4.4)$

Chapter 4: Extensive form

Strategic form: (Normal form)

Players move simultaneously,
represented by matrices/bimatrices

Extensive form:

Players move alternatively,
represented by game tree (directed tree)

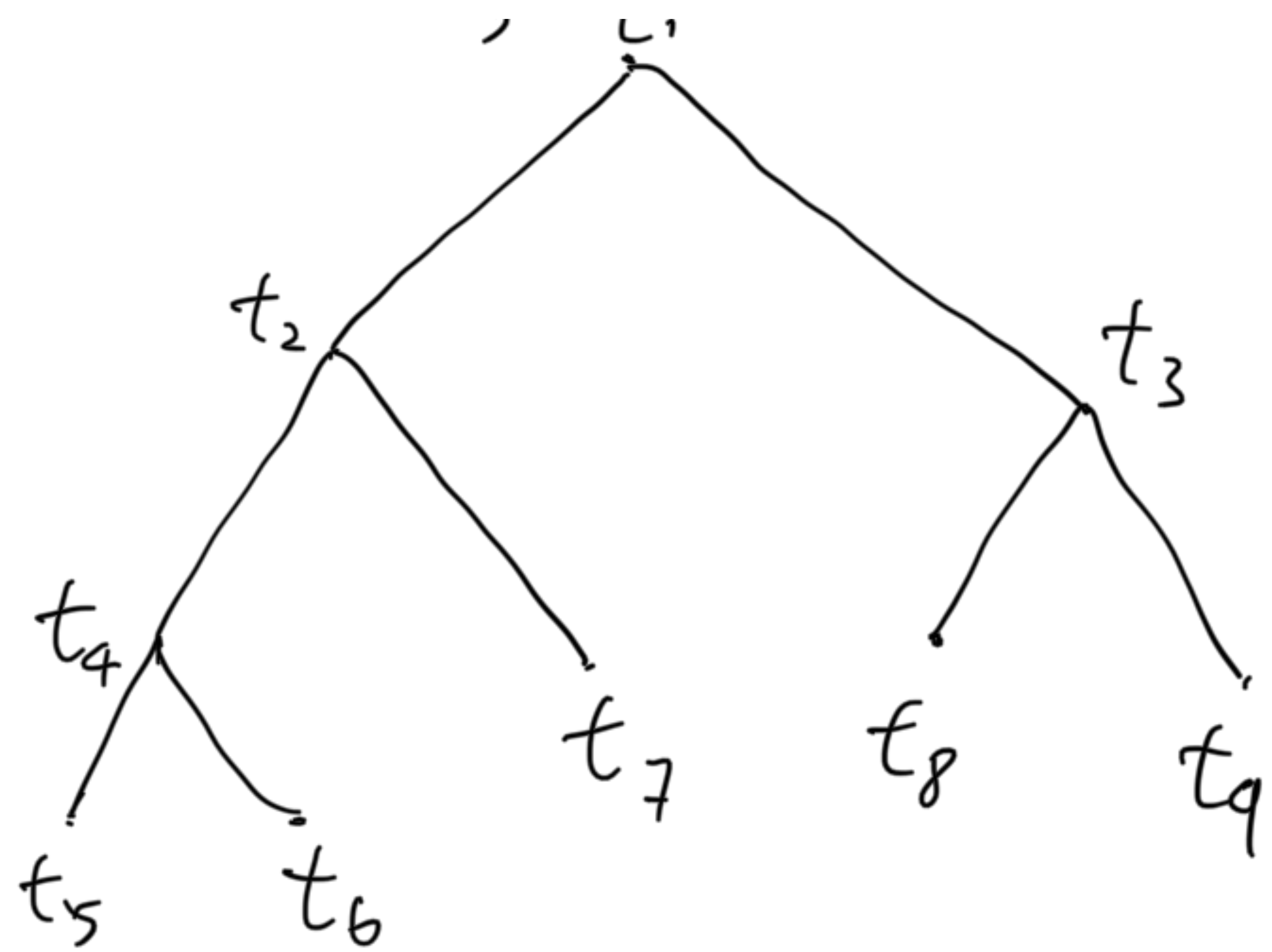
Directed graph:

(T, F): T is the set of vertices (nodes)

$$F : T \to \mathcal{P}(T), \quad \mathcal{P}(T) = \text{power set of } T$$
$$t \mapsto F(t) \qquad = \text{set of all subsets of } T.$$

$F(t) \subset T$ is the set of followers of t.

initial node $\longrightarrow$ t_0



$F(t_2) = \{t_4, t_7)$

$F(t_3) = \{t_8, t_9)$

(Directed) tree : There is an initial vertex t_0 such that for any $t \in T$, there is a unique path goes from t_0 to t.

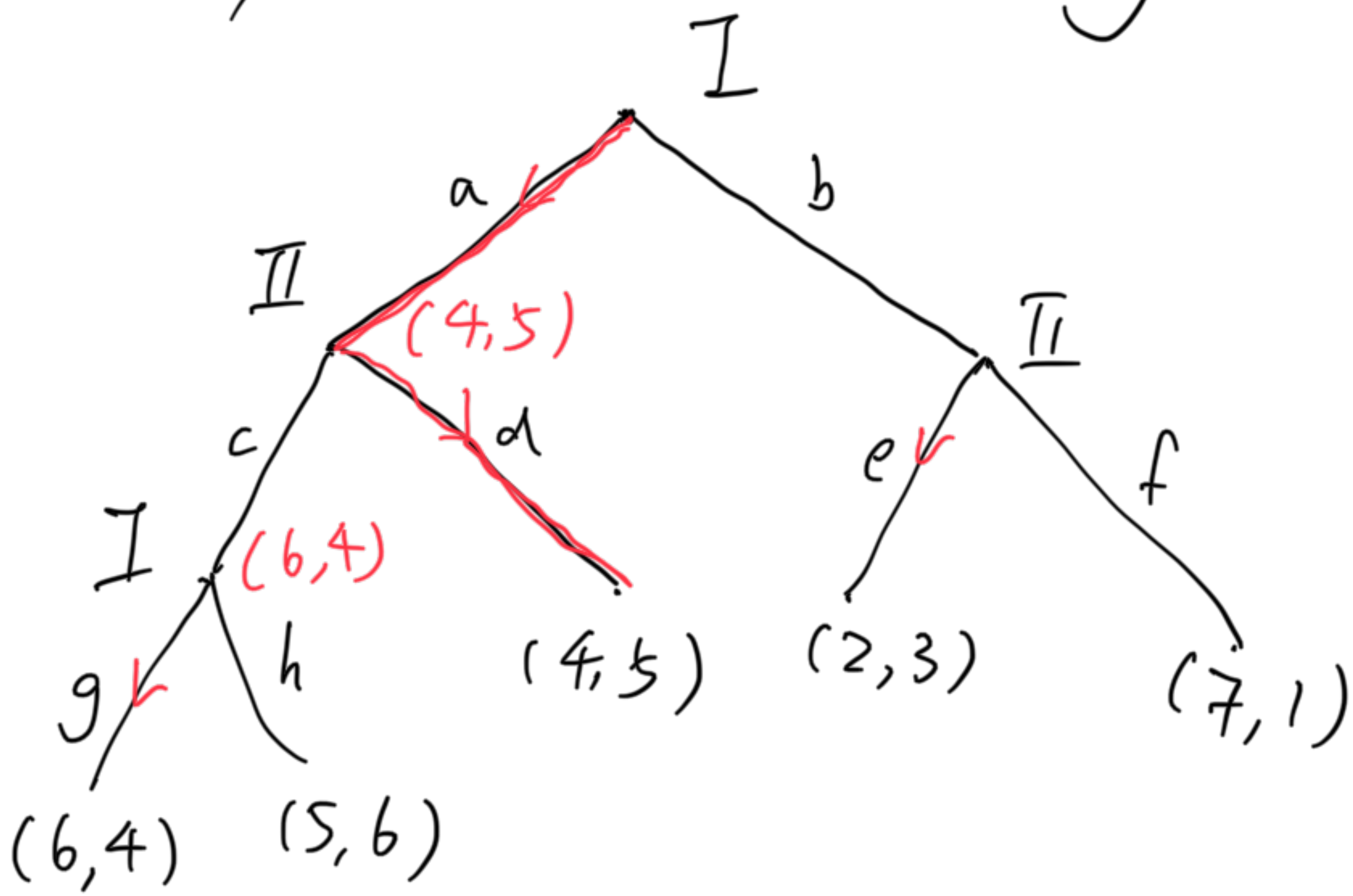
For two - person game, there are 3 types of nodes :

1. Node associated with player I : Branches of it correspond to moves of I.

2. Node associated with player II :

3. Terminal nodes ($F(t) = \phi$)

Payoff pair is assigned



Backward induction

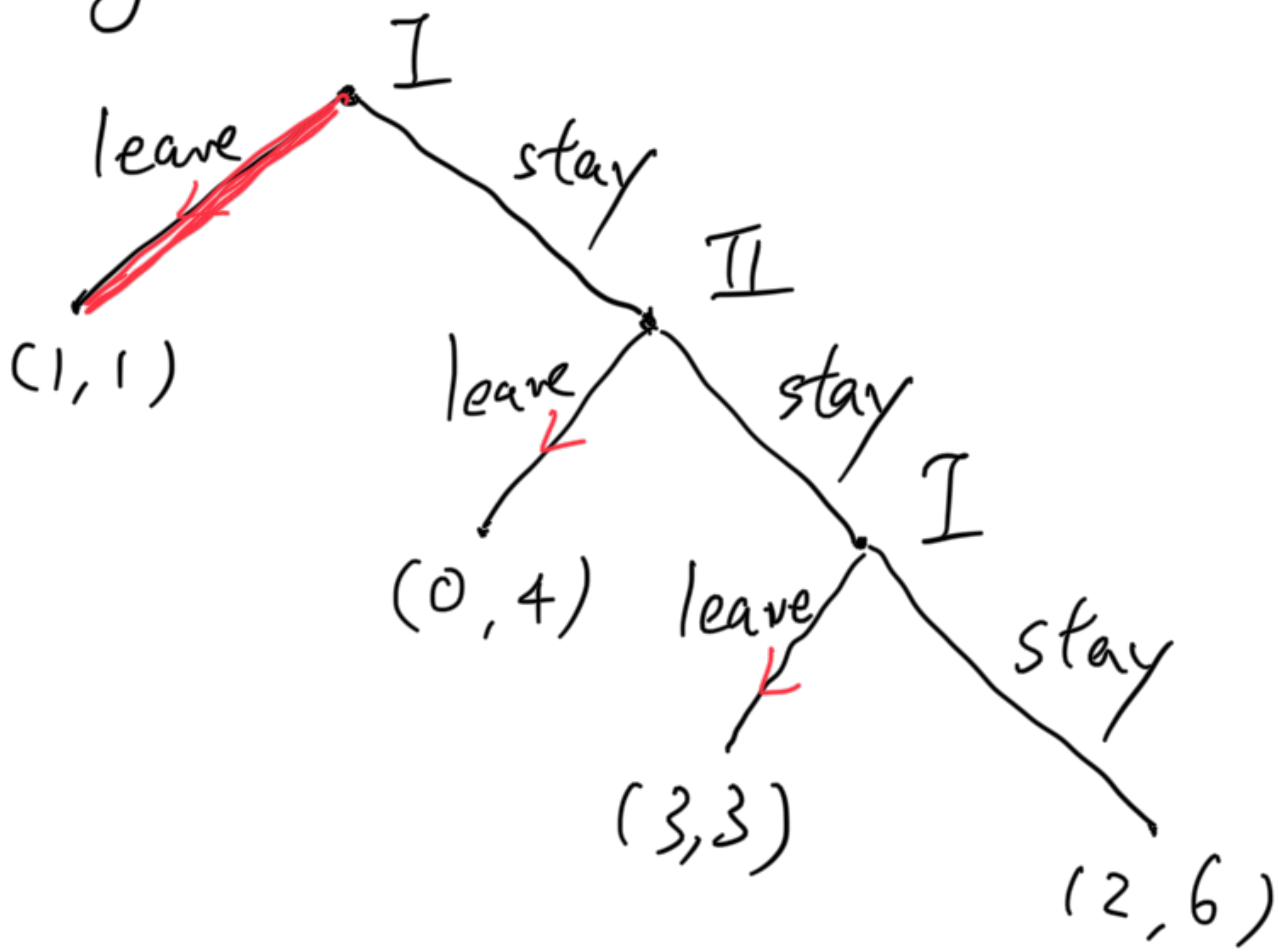
Centipede game :

Each player has $1 in his pile.

Choose Leave or Stay alternative

Stay: \$1 deducted from the player's pile and \$3 added to another player's pile.

The game ends if total amount reaches \$8.



Perfect information:

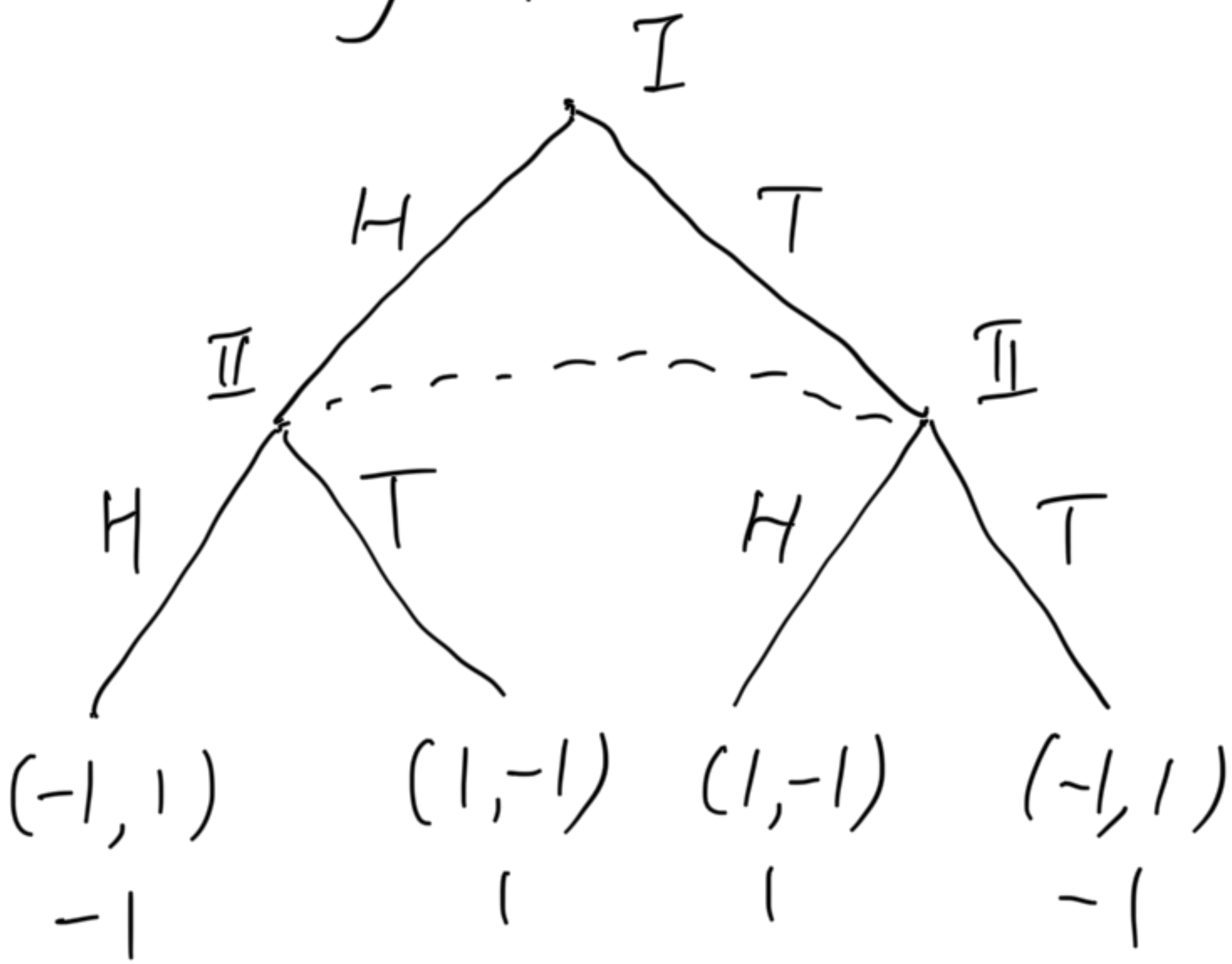
1. All players know all moves that have taken place

2. There is no chance moves.

Imperfect information:

1. Some moves are hidden.
2. There may be chance moves.

Matching pennies



Information set:

1. Two nodes are in the same information set if

the corresponding player does not know which node exactly the player is at.

2. Two nodes in the same information set are joined by dotted lines.
   (a) same number branches.
   (b) branches in one-to-one correspondeace

Perfect information: each information set contains only one node.

Extensive form: Kuhn tree

Strategies of players:

Choosing one branch for each information set

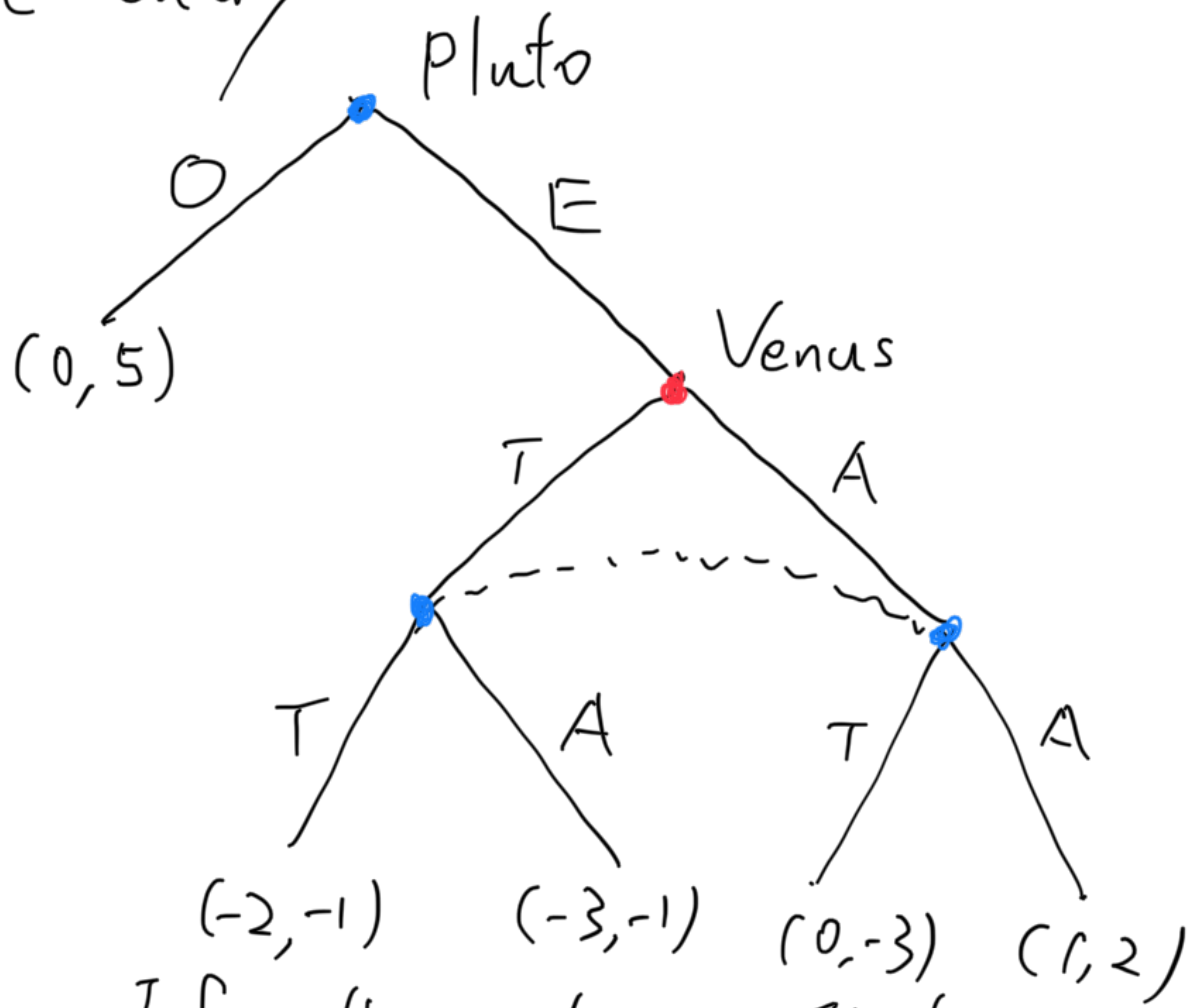
Number of strategies = product of number of branches of all information set.

Matching pennies: Player I and II

Matching pennies: Player I: H, T

Player II: H, T

Market entry:



	Information set	Strategies
Pluto	Initial, Enter	OT, OA, ET, EA
Venus	Enter	T, A

Remark: OT and OA look redundant but they are considered as two different strategies in the extensive form of game.

Strategic form (game bimatrix)

		Venus	
		T	A
Pluto	OT	(0,5)	(0,5)
	OA	(0,5)	(0,5)
	ET	(-2,-1)	(0,-3)
	EA	(-3,-1)	(1,2)

Three Nash equilibria:

(OT, T), (OA, T), (EA, A)